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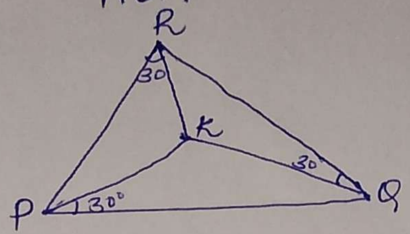
NO: 1



Given that $\angle KRP = \angle KQR = \angle KPQ = 30°$
In the triangle PQR, the sum of the angles is 180°
So $\angle KRP$ and $\angle KQR$ are both 30°
and $\angle KRP + \angle KQR = 30° + 30°$ (Angle sum)
$= 60°$

Hence $\angle PQR = 180° - 60°$
$= 120°$
Similarly $\angle KPQ$ and $\angle KRP$ are also both 30°
And $\angle KPQ + \angle KRP = 60°$ (Angle sum).
So $\angle PRQ = 180° - 60°$
$= 120°$
Then since both $\angle PQR$ and $\angle PRQ$ are 120° each
and the sum of the angles in a triangle is 180°
The $\angle PQR$ must also be 120°
∴ the three angles of a triangle PQR are 120° each

So since all the three angles of a triangle PQR are 120° each
Then the triangle PQR is an equilaterial triangle.

NO 2

ⓐ $\Lambda(G) = \sum_{g\in G} \operatorname{ord}(g)^{-m}$

Then $\Lambda(Z_n) = \sum_{g\in G} (K-m)$ for $K \in Z_n$

Since $\operatorname{ord}(g) \leq n$ (as $g \in G$ and $|G| = n$)

Then $\operatorname{ord}(g)^{-m} \leq n-m$

Now $\sum_{g\in G} \operatorname{ord}(g)^{-m} \leq \sum_{g\in G} (n-m)$

$\Lambda(G) \leq n^2 - nm$

Similarly for Z_n, we have

$\sum (K-m) = \frac{n(n-1)}{2} - mn$

$\Lambda(Z_n) = \frac{(n-1)n}{2} - mn$

Since $n^2 - mn \geq \frac{n(n-1)}{2}$

Then $\Lambda(G) \geq \Lambda(Z_n)$ As required

(b) Suppose $\Lambda(G) = \Lambda(Z_n)$, then $\sum_{g\in G} \operatorname{ord}(g)^{-m} = \sum_{g\in G} (K-m)$

As $\operatorname{ord}(g) \leq n$

We have $\sum_{g\in G} \operatorname{ord}(g)^{-m} \leq \sum K = \frac{n(n-1)}{2}$

Equality holds only when $\operatorname{ord}(g) = K$ for each $g \in G$

$\Leftarrow$ Conversely, if G and Z_n are isomorphic, then $\exists$ a bijective hormomorphism $\psi : G \longrightarrow Z_n$

Then $\operatorname{ord}(g) = \operatorname{ord}(\psi(g)) = \psi(g)$

$\sum_{g\in G} \operatorname{ord}(g) = \sum_{g\in G} \psi(g) = \sum K$

Now subtracting nm from both sides give

$\Lambda(G) = \Lambda(Z_n)$

$\therefore \Lambda(G) = \Lambda(Z_n)$ if and only if G and Z_n are isomophic.

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NO 3

$$\sum_{i=1}^{\infty} \frac{a^i}{(1+a)^i} = \sum \left(\frac{a}{1+a}\right)^i$$

This is a geometric series with the first term $\left(\frac{a}{1+a}\right)$ and common ratio $\left(\frac{a}{1+a}\right)$

For the series to converge uniformly on the interval $[0,1]$ We need to check the following conditions

(i) point wise convergence on $[0,1]$
For $a \in [0,1]$, the series converges since $\left|\frac{a}{1+a}\right| < 1$

(ii) Uniform convergence
We need to show that the series converges uniformly on $[0,1]$
We use the weierstrass M-test for this
let $M_i = \sup\left\{\left(\frac{a}{1+a}\right)^i : a \in [0,1]\right\}$
Then $M_i = \frac{1}{2^i}$ (since the maximum value occurs at $a=1$)
Since $\sum \frac{1}{2^i}$ converges, the orginal series converges uniformly on $[0,1]$ by the Weierstrass M-test

Therefore, the series $\sum_{i=1}^{\infty} \frac{a^i}{(1+a)^i}$ converges uniformly on the interval $[0,1]$